Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

8325226712

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

© UCLES 2023

1 Use logarithms to solve the equation $12^x = 3^{2x+1}$. Give your answer correct to 3 significant figures. [4]

2 A curve has equation $y = \dfrac{2 + 3\ln x}{1 + 2x}$.

Find the equation of the tangent to the curve at the point $\left(1, \frac{2}{3}\right)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

3 It is given that $\int_0^a (3e^{2x} - 1)\,dx = 12$, where a is a positive constant.

(a) Show that $a = \frac{1}{2}\ln\left(9 + \frac{2}{3}a\right)$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Use an iterative formula, based on the equation in **(a)**, to find the value of a correct to 4 significant figures. Use an initial value of 1 and give the result of each iteration to 6 significant figures. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

4 The polynomial $p(x)$ is defined by

$$p(x) = 2x^3 + 3x^2 + kx - 30,$$

where k is a constant. It is given that $(x - 3)$ is a factor of $p(x)$.

(a) Find the value of k. [2]

.....

.....

.....

.....

.....

(b) Hence find the quotient when $p(x)$ is divided by $(x - 3)$ and factorise $p(x)$ completely. [3]

.....

.....

.....

.....

.....

.....

.....

.....

(c) It is given that a is one of the roots of the equation $p(x) = 0$.

Given also that the equation $|4y - 5| = a$ is satisfied by two real values of y, find these two values of y. [3]

.....

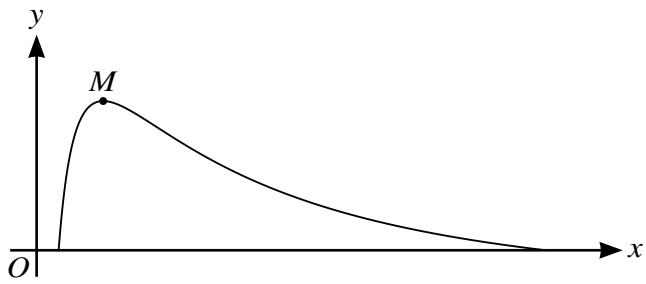
.....

.....

.....

.....

5



The diagram shows the curve with parametric equations

$$x = 4e^{2t}, \qquad y = 5e^{-t}\cos 2t,$$

for $-\frac{1}{4}\pi \leqslant t \leqslant \frac{1}{4}\pi$. The curve has a maximum point M.

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the coordinates of M, giving each coordinate correct to 3 significant figures. [5]

6 Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \left(4\cos^2 2x + \frac{1}{\cos^2 x}\right) \mathrm{d}x = \frac{3}{4}\sqrt{3} + \frac{1}{6}\pi - 1.$ [7]

7 **(a)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Solve the equation $7\cos\theta + 24\sin\theta = 18$ for $0° < \theta < 360°$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) As β varies, the greatest possible value of

$$\frac{150}{7\cos\frac{1}{2}\beta + 24\sin\frac{1}{2}\beta + 50}$$

is denoted by V.

Find the value of V and determine the smallest positive value of β (in degrees) for which the value of V occurs. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2023